ON CONTINUOUS-TIME TWO PERSON FULL-INFORMATION BEST CHOICE PROBLEM WITH IMPERFECT OBSERVATION

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SUMMARY. A zero-sum game version of the continuous-time full-information best choice problem is considered. Two players observe sequentially a stream of i.i.d random variables from a known continuous distribution appearing according to some renewal process with the object of choosing the largest one. The horizon of observation is a positive random variable independent of observations. The observations of the random variables are imperfect and the players are informed only whether it is greater than or less than some levels specified by both of them. The normal form of the game is derived. For the Poisson stream and the exponential horizon the value of the game and the form of the optimal strategy are obtained. It is worth to emphasize the difference from of solution of the game for the various relation of the intensity of the Poisson stream and the parameter of the exponential horizon.

1. INTRODUCTION

The paper deals with the following zero-sum game version of the continuous-time full-information best choice problem. Two players observe sequentially a stream of i.i.d random variables from a known continuous distribution appearing according to some renewal process with the object of choosing the largest. Decision about choosing must be made before a moment $T$, which is a positive random variable independent of observations. The random variables cannot be perfectly observed. Each time a random variable is sampled the sampler is informed only whether it is greater than or less than some level specified by him. Players can choose at most one observation. After each sampling players take a decision for acceptance or rejection of the observation. If both want to accept the same observation the priority is given to the specified player, say Player 1. The class of adequate strategies and the suitable gain function for the problem


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is constructed. The natural case of the Poisson renewal process with parameter \( \lambda \) and exponentially distributed \( T \) with parameter \( \mu \) is examined in details. The value of the game and the optimal strategy depends on \( \lambda \) and \( \mu \) by \( r = \frac{\mu}{\mu + \lambda} \) only. The optimal strategy for Player 1 is always pure. The form of the second player's strategy depends on the value of \( \mu \) and \( \lambda \). It is pure for \( \mu \gg \lambda \). For \( \lambda \gg \mu \) it is a mixture of two pure strategies. It is the first zero-sum game extension of the best choice problem which has the randomized optimal strategy for some values of the parameters \( \mu \) and \( \lambda \). It seems the consequence of the restricted knowledge of the realization of the stopping process. On the other hand such imperfect observations appear in some destructive testing when it is very difficult or expensive to change the specified level of strength. So the randomized strategies are not easy applied. The obtained result can explain how to avoid the unpleasant situation (difficulties) in some competitive experiments.

2. The priority game with imperfect observation

Let \( X_1, X_2, X_3, \ldots \) be a sequence of i.i.d random variables with common and known continuous distribution \( F \) defined on a probability space \( (\Omega, \mathcal{F}, P) \). \( X \)'s appear according to some renewal process and let \( \rho_n \) stand for the length of time interval between \( X_{n-1} \) and \( X_n \) (for convenience is assumed \( X_0 = 0 \) by definition), i.e. \( \rho_1, \rho_2, \rho_3, \ldots \) are i.i.d positive random variables with a continuous distribution \( G \). A positive random variable \( T \) with a distribution \( H \) represents the moment when the observation will be terminated. The \( X \)'s, \( \rho \)'s and \( T \) are independent.

The sequence of random variables is sequentially sampled one by one by two decision makers (players). However the observations are imperfect and the exact realized values are not known. Players specify only their level of sensitivity (impressionability) and they are able to know whether the observed random variable is greater than or less than prescribed levels, chosen themselves. After \( X_n \) is observed Player 1 has to accept or reject the observation. If he rejects it in turn, Player 2 has to decide if he rejects the observed realization. One can say that Player 1 has priority to accept the realization. When some player accepts the observation, then the other one will investigate the sequence of future realizations having opportunity to accept one of them. Neither recall nor uncertainty of selection is allowed. The aim of the players is to choose the best observation (the maximal one). The review of the related problems and results for discrete time one can find in papers by Sakaguchi (1984) and Porosiński (1991). Let us mention here the papers concerning variation of the best choice problem with Poisson arrivals considered by Cowan and Zabczyk (1978) and Enns and Perenstein (1990).

In this paper we admit that the problem is modelled by two person zero-sum game. The similar models for the no-information case have been considered e.g.
in Szajowski (1993) and Enns and Ferenstein (1987). The structure of strategy sets and the form of the gain functions are different in these problems.

Let

\[ S_n = \rho_1 + \ldots + \rho_n, \quad n = 1, 2, \ldots, S_0 = 0, \quad \ldots (1) \]

\[ N(t) = \max\{n \geq 0 : S_n \leq t\}, \quad t \geq 0. \quad \ldots (2) \]

So \( S_n \) is the waiting time of the \( n \)-th observation and \( N(t) \) is the total number of \( X \)'s that appeared up to the time \( t \). At the moment when \( X_n \) is observed, all previous values of \( X \)'s and \( \rho \)'s are known and moreover it is known whether the moment \( T \) follows or not i.e. the \( \sigma \)-field of information is

\[ \mathcal{F}_n = \sigma \{ X_1, \ldots, X_n, \rho_1, \ldots, \rho_n, X\{T > S_1\}, \ldots, X\{T > S_n\} \}, \quad n = 1, 2, \ldots, \]

where \( \chi_A \) stands for the indicator function of the event \( A \). Let \( \mathcal{S} \) be the set of stopping times with respect to \( \{\mathcal{F}_n\}_{n=1}^{\infty} \). Since the observations are imperfect we admit \( \mathcal{S}_0 = \{ \tau \in \mathcal{S} : \tau = \inf\{n \leq N(T) : X_n \geq x\}, x \in \mathcal{R} \} \) as the class of adequate strategies for one person decision problem. We assume that the players have \( \mathcal{S}_0 \) as the sets of strategies and the above described priority of Player 1 will be involved in the gain function.

Let Player 1 and Player 2 choose the levels \( x \in \mathcal{R} \) and \( y \in \mathcal{R} \), respectively. If \( x < y \) and Player 1 accepts, \( X_x \) then he gets +1 when \( \{N(T) \geq s, X_1 < x, \ldots, X_{s-1} < x, X_s \geq x, X_{s+1} < X_s, \ldots, X_{N(T)} < X_s\} \), −1 when there is \( t > s \) such that \( \{N(T) \geq t, X_1 < x, \ldots, X_{s-1} < x, X_s \geq x, X_{s+1} < y, \ldots, X_{t-1} < y, X_t > X_s \vee y, X_{t+1} < X_t, \ldots, X_{N(T)} < X_t\} \) and 0 otherwise. Let \( x \geq y \). In this case if Player 1 accepts \( X_x \) then he gets +1 when \( \{N(T) \geq s, X_1 < y, \ldots, X_{s-1} < y, X_s \geq x, X_{s+1} < X_s, \ldots, X_{N(T)} < X_s\} \) or there is \( t > s \) such that \( \{N(T) \geq t, X_1 < y, \ldots, X_{s-1} < y, X_s \geq x, X_{s+1} < x, \ldots, X_{t-1} < x, X_t \geq x, X_{t+1} < X_t, \ldots, X_{N(T)} < X_t\} \), −1 when there is \( t > s \) such that \( \{N(T) \geq t, X_1 < y, \ldots, X_{s-1} < y, X_s \geq x, X_{s+1} < y, \ldots, X_{t-1} < y, X_t > X_s, X_{t+1} < X_t, \ldots, X_{N(T)} < X_t\} \) or \( \{N(T) \geq s, X_1 < y, \ldots, X_{s-1} < y, y \leq X_s < x, X_{s+1} < x, \ldots, X_{t-1} < x, X_t \geq x, X_{t+1} < X_t, \ldots, X_{N(T)} < X_t\} \) and 0 otherwise. Taking into account the above consideration we can assume that the observed random variables have the uniform distribution and the sets of strategies \( \mathcal{S}_0 \) is equivalent to the interval \([0, 1]\). It reduces the problem to the zero-sum game on the unit square. The payoff function is the expected value of the described Player 1’s game.

3. EQUIVALENT GAME ON UNIT SQUARE

Let \( f_N(x, y) \) stand for the payoff of Player 1 when the levels \((x, y)\) are chosen
and \( N(T) = N \), Then for \( x < y \) we have

\[
\begin{align*}
 f_N(x, y) &= \sum_{s=1}^{N} x^{s-1} \int_{x}^{1} x_s^{N-s} dx_s - \sum_{t=2}^{N} \sum_{i=1}^{t-1} \left( \int_{x}^{y} y^{t-s-1} dx_s \int_{y}^{1} x_t^{N-t} dx_t \right) \\
+ \int_{y}^{1} y^{t-s-1} dx_s \int_{x}^{1} x_t^{N-t} dx_t &= \sum_{i=1}^{N} \frac{x^{N-i} - x^N}{i} \\
- \sum_{j=1}^{N-1} \sum_{i=j+1}^{N} \frac{x^{N-i} y^{i-j-1}}{j} \left( 1 - x - (y - x)y^j - \frac{1 - y^{j+1}}{1 + j} \right) 
\end{align*}
\]

and for \( x \geq y \)

\[
\begin{align*}
 f_N(x, y) &= \sum_{s=1}^{N} y^{s-1} \int_{x}^{1} x_s^{N-s} dx_s + \sum_{s=1}^{N-1} \sum_{t=s+1}^{N} y^{s-1} \int_{y}^{1} x_t^{t-s-1} dx_s \int_{x}^{1} x_t^{N-t} dx_t \\
- \sum_{s=1}^{N} y^{s-1} \int_{y}^{1} x^{N-s} dx_s \sum_{t=s+1}^{N} y^{t-s-1} dx_s \int_{x}^{1} x_t^{N-t} dx_t \\
= \sum_{i=1}^{N} \frac{y^{N-i}}{i} (1 - 2x^i + y^i) \\
+ \sum_{j=1}^{N-1} \sum_{i=j+1}^{N} \left( y^{N-i} x^{i-j-1}(x - y) \frac{1 - x^i}{j} - \frac{y^{N-j-1}}{j} \left( 1 - x - \frac{1 - x^{j+1}}{j + 1} \right) \right).
\end{align*}
\]

The payoff function is then \( f(x, y) = \sum_{N=0}^{\infty} f_N(x, y)P(N(T) = N) \), where \( f_N(x, y) \) is given by (3).

Based on the distributions of \( T \) and process \( N(t) \) the distribution of total number of observation can be found.

\[
P(N(T) = N) = \int_{0}^{\infty} P(S_N \leq t, S_{N+1} > t) dH(t) \\
= \int_{0}^{\infty} dH(t) \int_{0}^{t} P(S_{N+1} > t | S_N = s) dG^*N(s). \quad \ldots (4)
\]

\[
= \int_{0}^{\infty} dH(t) \int_{0}^{t} P(pN + 1 > t - s) dG^*N(s),
\]

where \( G^*N \) stands for a distribution of \( S_N \).

Due to the form of the payoff function, it is very difficult to obtain the optimal levels \((x, y)\) explicitly, even if the distribution \( G \) and \( H \) are fixed. Nevertheless in a natural case considered below the solution has a very simple form.
TABLE 1. THE SOLUTION OF THE GAME WITH THE POISSON STREAM 
OF OPTIONS WITH PARAMETER $\lambda$ AND THE 
EXPONENTIAL HORIZON WITH PARAMETER $\mu$.

<table>
<thead>
<tr>
<th>$r = \frac{r}{r+1}$</th>
<th>Player 1</th>
<th>Player 2</th>
<th>Value of the game</th>
<th>Probability of success for Player 1 : $r$.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s^*(r)$</td>
<td>$\alpha^*(r)$</td>
<td>$\beta^*(r)$</td>
<td>$\gamma^*(r)$</td>
<td>$u(r)$</td>
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<tr>
<td>0.2030</td>
<td>0.3702</td>
<td>0.7555</td>
<td>0.2030</td>
<td>0.5327</td>
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<tr>
<td>0.21</td>
<td>0.3703</td>
<td>0.7550</td>
<td>0.21</td>
<td>0.5328</td>
</tr>
<tr>
<td>0.22</td>
<td>0.3708</td>
<td>0.7539</td>
<td>0.22</td>
<td>0.5331</td>
</tr>
<tr>
<td>0.23</td>
<td>0.3717</td>
<td>0.7523</td>
<td>0.23</td>
<td>0.5334</td>
</tr>
<tr>
<td>0.24</td>
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<td>0.7502</td>
<td>0.24</td>
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<tr>
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<td>0.7476</td>
<td>0.25</td>
<td>0.5350</td>
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<tr>
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<td>0.7439</td>
<td>0.26</td>
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</tr>
<tr>
<td>0.27</td>
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<td>0.7393</td>
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<tr>
<td>0.28</td>
<td>0.3811</td>
<td>0.7331</td>
<td>0.28</td>
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</tr>
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<tr>
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<td>0.45</td>
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<td>0.56</td>
<td>0.6440</td>
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<td>0.0002</td>
<td>0.5671</td>
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</tr>
<tr>
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<td>0.5700</td>
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</tr>
<tr>
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</tr>
<tr>
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<td>0.5900</td>
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<tr>
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<td>0.6704</td>
</tr>
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<td>1.0000</td>
</tr>
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</table>

Theorem 4.1. Let $F(x, y)$ be continuous function on the unit square $I \times I$ and let $F(x, y)$ be concave in $x$ for each $y$. Then the zero-sum game $\Gamma = (I, I, F(x, y))$ has a saddle point of the form $(\delta_s, Q^*(y))$, where $Q^*(y) = \alpha \delta_s(y) + (1 - \alpha)\delta_d(y)$ for some $0 \leq a, c, d, \alpha \leq 1$.

Let us consider the function $g(s, t)$ on the unit square. It is continuous and concave on $s$ for every fixed $t$. For every fixed $s$ it has two local minima on $t, 0 \leq t_1(s) \leq s$ and $s \leq t_2(s) \leq 1$ for every fixed $s$. The game with the gain function $g(s, t)$ considered on unit square has the following solution. Player 1 has an optimal pure strategy $s^* \approx 0.3702$ and Player 2 has an optimal mixed strategy $Q^*(t) = \alpha \delta_1(t) + (1 - \alpha)\delta_2(t)$, where $\alpha^* \approx 0.7555, t_1^* \approx 0.2030, t_2^* \approx 0.5327$. The value of the game $val = \max_s \min_t g(s, t) \approx 0.2030$. The optimal strategy for Player 1 is obtained as the unique $s^*$ such that $\min_t g(s^*, t) = \max_s \min_t g(s, t)$. The parameters $\alpha^*, t_1^*, t_2^*$ of the best strategy for Player 2 are obtained from conditions.
4. The Poisson Stream of Options

Let \( G \) be exponential with the parameter \( \lambda \). Thus \((N(t))_{t \in [0, +\infty)}\) is the Poisson process with the parameter \( \lambda \). Moreover, let \( T \) have the exponential distribution with the parameter \( \mu \). In this case we have for \( s \geq 0 \)

\[
\frac{\lambda^N}{(N-1)!} s^{N-1} e^{-\lambda t} ds
\]

and the probability that the exactly \( N \) observations appear up the time \( T \) given by (4) can be calculated as

\[
P(N(T) = N) = \int_0^\infty \left( \int_0^t e^{-\lambda(t-s)} \frac{\lambda^N}{(N-1)!} s^{N-1} e^{-\lambda s} ds \right) \mu e^{-\mu t} dt
\]

\[
= \int_0^\infty \frac{\lambda^N}{N!} \mu t^{N} e^{-(\lambda+\mu)t} dt = \frac{\mu \lambda^N}{(\lambda + \mu)^{N+1}}.
\]

The payoff function \( f(x, y) \) can be written, after simplifications, as a function \( g(s, t) \) of new coordinate variables \( s = \mu/(\mu + \lambda(1-x)), t = \mu/(\mu + \lambda(1-y)) \),

\[
g(s, t) = \begin{cases} g_1(s, t) & \text{if } r \leq t \leq s \\ g_2(s, t) & \text{if } r \leq s < t \leq 1, \end{cases}
\]

where

\[
g_1(s, t) = t \ln t - s \ln s + \frac{t^2}{s} - \frac{t^2}{s} - t s - t^2
\]

\[
g_2(s, t) = t \ln t - s \ln s - t s - t^2,
\]

and \( r = \frac{\mu}{\mu + \lambda} \). This transformation keeps monotonicity. It is very interesting and important that the gain function \( g(s, t) \) depends on \( \lambda \) and \( \mu \), through relation \( \frac{\mu}{\lambda + \mu} \), by its domain only.

By the above consideration the game is transformed to the zero-sum game on \([r, 1] \times [r, 1]\) with the gain function given by (7). This function is continuous of both variables and concave on \( s \). The existence and a form of optimal strategies for such game can be found in Parthasarathy and Raghavan (1975), where the generalization of well known theorem of Bohnenblust, Karlin and Shapley (1950) (see also Radzik (1993)) is given. Denote \( \delta_x \) the probability distribution function degenerated on \( x \).
\[ g(s^*, t_1) = g(s^*, t_2) = \text{val} \]
\[ \frac{\partial g_1(s^*, t_1)}{\partial s} \leq 0 \leq \frac{\partial g_2(s^*, t_2)}{\partial s} \]
\[ \alpha \frac{\partial g_1(s^*, t_1)}{\partial s} + (1 - \alpha) \frac{\partial g_2(s^*, t_2)}{\partial s} = 0. \]

The obtained solution on \([0, 1] \times [0, 1]\) is also valid on \([r, 1] \times [r, 1]\) for \(r \leq t^*_1\). For \(r > t^*_1\) the players have to modify their strategies according to the above described conditions applied on \([r, 1] \times [r, 1]\).

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**Fig. 1.** The value of the game and the probability of success for Player 1 for the game with the Poisson stream of options with parameter \(\lambda\) and the exponential horizon with parameter \(\mu, (r = \mu/\mu + \lambda)\). For the main problem, based on the auxiliary game, we can formulate

**Proposition 4.2.** For exponential \(T\) with parameter \(\mu\) and \(\rho\)'s with parameter \(\lambda\) there exists a solution of the game. It has the following form depending on \(\lambda\) and \(\mu\):

1. If \(\mu \leq \lambda \frac{1 - t^*_1}{1 - t^*_1}\) then the optimal level for Player 1 is \(x^* = 1 - \frac{\mu(1 - s^*)}{\lambda s^*}\), where \(s^* \approx 0.3702\). Player 2 is using the mixed strategy \(Q^*(y)\) with \(\alpha^* \approx 0.78555\), \(y_1^* = 1 - \frac{\mu(1 - t^*_1)}{\lambda t^*_1}\) and \(y_2^* = 1 - \frac{\mu(1 - t^*_1)}{\lambda t^*_1}\), where \(t^*_1 \approx 0.2030\) and \(t^*_2 \approx 0.5327\). The value of the game \(\text{val} \approx 0.2030\) and is independent of \(\mu\) and \(\lambda\).

2. If \(\lambda \frac{t^*_1}{1 - t^*_1} < \mu \leq \lambda \frac{1}{1 - p_1}\), where \(p_1 \approx 0.5671\) then the optimal levels for Player 1 are \(x^*(\mu, \lambda) = 1 - \frac{\mu(1 - s^*(r))}{\lambda s^*(r)}\), where \(s^*(r)\) is the solution of the equation \(v(r) = \max_{s \in [r, 1]} \min_{t \in [r, 1]} g(s, t)\). The optimal strategy \(Q^*(y)\)
for Player 2 has $y_1^*(\mu, \lambda) = 1 - \frac{\mu(1-t^*(r))}{\lambda^p(r)}$ and $y_2^*(\mu, \lambda) = 1 - \frac{\mu(1-t^*(r))}{\lambda^q(r)}$, where $t^*_i(r) = r$, $\alpha^*(r)$ and $t^*_2(r)$ fulfill (8) on $[r, 1] \times [r, 1]$.

3. If $\mu > \lambda \frac{p^*}{1-p^*}$, then the saddle point is in pure strategies and has the form $(0, y^*(\mu, \lambda))$. The strategy $y^*(\mu, \lambda) = 1 - \frac{\mu(1-t^*(r))}{\lambda^p(r)}$, where $t^*(r)$ fulfills relations $g(r, t^*(r)) = v(r)$ and $\frac{\partial g(r, t^*(r))}{\partial s} \leq 0$.

The probability of success $P_r$ for Player 1 when both players are using the optimal strategies is $P_r = -s^*(r) \ln s^*(r)$ in all above cases.

It is interesting and quite unexpected that in all natural situations (i.e. when $\mu << \lambda, r \leq t^* \approx 0.2030$) the value of the game is constant (see also Figure 1 and Table 1). It is not surprising that the probability of success $P_r = -s^*(r) \ln s^*(r)$ for Player 1 is independent of the second player's decision for two person model. For $r \leq t^*_i$ the probability of success $P_r = -s^* \ln s^* \approx 0.3679$ for Player 1 is only a little less than that for one person model ($= e^{-1}$, see geometric discrete model in Porosiński (1991)).

References


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